Homeopathy (2003) 92, 152–160

doi:10.1016/S1475-4916(03)00038-9, available online at www.sciencedirect.com

EDUCATION AND DEBATE

Patient–practitioner–remedy (PPR) entanglement. Part 3. Refining the quantum metaphor for homeopathy

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The notion of patient–practitioner–remedy (PPR) entanglement, previously proposed for homeopathy, is refined by adapting concepts derived from Greenberger, Horne, and Zeilinger's treatment of three-particle entanglement (GHZ states), and a generalised version of quantum theory, called weak quantum theory (WQT). These suggest that for maximum PPR entanglement during the therapeutic encounter, the practitioner's awareness needs to be directed inward as well as outward toward the patient, and that health and disease are mirror images of each other, similar to and represented by, the relationship of complex numbers to their complex conjugates. *Homeopathy* (2003) 92, 152–160.

Keywords: non-locality; three-particle GHZ entanglement; PPR entanglement; weak quantum theory; quantum metaphors for homeopathy

Introduction

Non-locality is perhaps *the* defining concept of quantum theory and, as previous papers in this series have attempted to show, might usefully illuminate the nature of the triadic relationship between patient, practitioner, and remedy.[1–3] I have termed this patient–practitioner–remedy (PPR) entanglement by analogy with Erwin Schrödinger's notion of entanglement between correlated parts of a quantum system based on a gedanken (or thought) experiment of Einstein, Podolsky, and Rosen (EPR).

Non-locality has been defined as 'the mysterious ability of Nature to enforce correlations between separated but entangled parts of a quantum system even if they are out of speed-of-light contact with each; and to reach instantaneously across vast spatial distances or even across time itself, to ensure that the parts of a quantum system are made to match'.[4] The implications of non-locality for the scientific world-view are quite staggering. For example, non-locality forces us to conclude that at its most primary and basic level, there is an undivided wholeness about the seemingly separate parts of physical reality whose existence can be inferred, but not proven by experiment. In coming to terms with the non-local view of the universe, it is being found necessary to abandon the three-century-old division between mind and matter, sanctioned by classical physics.[5]

By previously treating the PPR relationship in the context of a non-local entangled triad, and utilising as a metaphor the transactional interpretation of quantum mechanics, I have modelled certain observations about the homeopathic process, including homeopathic aggravations, and developed a concept of miasms based on the action of disease and susceptibility outside of time.[2] Then, using a different model based on the molecular quantum theory (MQT) of tri-atomic molecules, the quantum metaphor for

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Received 13 January 2003; revised 13 March 2003; accepted 7 April 2003

homeopathy was extended and used to suggest that the more attenuated the remedy, the greater the potential for PPR entanglement, and hence a curative outcome. *This is a prediction reminiscent of classical homeopathic thinking, in which it is considered that the more potentised the remedy, the deeper the level of cure achievable.*[3]

Because they are mainly qualitative and metaphorical in nature, these quantum theoretical models are somewhat crude. MQT effectively disentangles the nuclear and electronic components of molecules from each other (except in a molecule's electronic ground state where electrons with opposing spins in the same orbital are entangled).[6] Although this strategy enables chemists to perform calculations that predict molecular energy states and conformations sometimes with a stunning degree of accuracy, MQT must have stringent limitations to its metaphorical use in describing the PPR entangled state.

There is therefore a need for a better quantum paradigm to metaphorically (or otherwise) describe the PPR entangled state. In this respect, the Greenberger–Horne–Zeilinger (GHZ) state derived from the entanglement of three quantum particles (eg, photons or electrons) offers such as paradigm.[7] At the same time, it may be possible to describe the homeopathic process in a semi-quantitative manner, using a more general version of quantum theory (the so-called weak quantum theory or WQT)[8] in which certain axioms of quantum theory are relaxed, permitting application to non- or less physical contexts. This paper begins the exploration of these two avenues of thought, first by describing two- and three-particle (GHZ) entanglement and WQT, then showing how these ideas can illuminate certain features of the therapeutic process in homeopathy.

Quantum entanglement

Introduction

Almost 70 years after its realisation by Erwin Schrödinger, quantum entanglement is currently of practical interest because of its possible roles in the development of quantum computing, cryptography, and teleportation.[9]

Essentially, two or more interacting quantum entities can have entangled states, which have no parallel in classical physics. Thus, entangled quantum entities do not have their own individuality: only the entangled group as a whole has a well-defined state. It is also important to realise that entangled entities behave as if they were connected regardless of the distance between them: a measurement on one of the entities instantaneously provides information on its entangled partners no matter the intervening time or space between them. As a result, quantum entanglement is coming to be seen as a new type of physical resource (just as energy and information are physical resources), especially for solving problems of information processing (see above). And if quantum entanglement is a resource, then like matter and energy, *not only will it be limited, it ought also to be quantifiable.*[10]

Quantum entanglement must be differentiated from mere classical correlation. Thus two particles do not become entangled just because they have the same physical properties: they have to interact with each other in some way. Wootters uses the following analogy, '... ordinary correlation is like two people enjoying the same books—they do not need to have met... in order to enjoy them—whereas entanglement is more like being married.'[9]

Spin and polarisation

In purely physical terms, entanglement occurs between particles via their fundamental properties, eg, their spin if they are sub-atomic particles, or their polarisation if they are photons. Thus, for electrons, protons, and neutrons with spin of $\frac{1}{2}$, their spin directions are imagined as pointing either in the 'up' or 'down' directions: for photons, their polarisation is imagined as pointing in either the 'horizontal' or 'vertical' directions. One can distinguish between these two different states of spin or polarisation, but one cannot determine a particle's original spin or polarisation state. However, transformations from one state to the other can be achieved and one does not need to know the initial direction of spin or polarisation for such transformations to be carried out.

Let us consider a single spin-$\frac{1}{2}$ particle. Intuitively, its two different spin states are imagined as pointing in opposite directions in space, either 'up' or 'down'. Mathematicians describe such states in terms of two-dimensional complex space vectors, represented in Dirac notation as 'kets'. Thus, in terms of two of these space vectors, the kets $|\uparrow\rangle$ and $|\downarrow\rangle$ (where $\uparrow$ and $\downarrow$ represent the 'up' and 'down' spin of the particle), any other direction of spin can be written as a superposition of these 'up' and 'down' states, eg, $a|\uparrow\rangle + b|\downarrow\rangle$, where a and b are complex numbers (see appendix and footnote†) such that in terms of their real parts, $|a|^2+|b|^2 = 1$.[11] Maximal superposition occurs when $a = b$ and therefore, again in terms of their real parts, $|a| = |b| = \pm 1/\sqrt{2}$, yielding the maximal superposition spin state $1/\sqrt{2}\,(|\uparrow\rangle + |\downarrow\rangle)$ 'pointing to the right', while 'pointing to the left' is the maximal superposition spin state $1/\sqrt{2}\,(|\uparrow\rangle - |\downarrow\rangle)$. In other words, the total number of maximal superpositions possible with one particle is $2^1 = 2$.

†A complex number $x+iy$ is made up of a 'real' part, x, and an 'imaginary' part iy, which consists of a real number y multiplied by $i = \sqrt{-1}$, so that $i^2 = -1$. Taking the square of a complex number means multiplying it by what is called its complex conjugate, $x - iy$. Thus $(x+iy)(x-iy) = x^2 + xiy - xiy - i^2y^2 = x^2 - i^2y^2 = x^2 + y^2$ as $-i^2 = +1$: in other words, the square of a complex number is equal to the sum of the squares of its real parts.

Two- and three-particle entanglement: the GHZ state

Now let us consider what happens when two or more particles are entangled. First, the entangled particles have to be prepared. In the two-particle case, this occurs when an excited atom returns to its ground state by sequentially emitting two photons, or when two sub-atomic spin-$\frac{1}{2}$ particles, eg, protons, are scattered off each other, thereby entangling them.

According to classical physics, specifying the state of each of the individual particles should be enough to specify the state of the pair. Not so in quantum mechanics. In mathematical terms, a general pure state of two spin-$\frac{1}{2}$ particles, $|\Psi_{gen}\rangle$, has to be written as the superposition of four possible states: one from the entangled pair having both of their spins pointing 'up'; one from both spins pointing down; and two from the entangled pair having their spins pointing in opposite directions, ie, in terms of their space vectors, or kets:

$$|\Psi_{gen}\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle \quad (1)$$

As in the previous example of a single spin-$\frac{1}{2}$ particle a, b, c, d are complex numbers such that the sum of the squares of their real parts $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$.

There is a relationship connecting the complex numbers a, b, c, and d which provides a measure of the degree of entanglement between the two electrons, which is that the real parts of the differences of the products of the complex numbers $ad-bc$ must not equal zero, ie, $|ad-bc| > 0$, and the greater this difference, the greater the degree of entanglement. Only when $|ad-bc| = 0$, will states of the form shown in equation (1) be classically factorised into separate un-entangled states of the individual spin-$\frac{1}{2}$ particles (see Appendix).

For two entangled particles, there are a total of four (ie, 2^2) possible maximally entangled states, ie, when $|a| = |d| = 1/\sqrt{2}$, and $|b| = |c| = 0$, ie, $1/\sqrt{2}$ $(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)$; and when $|b| = |c| = 1/\sqrt{2}$, and $|a| = |d| = 0$, ie, $1/\sqrt{2}$ $(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, all of which are called Bell states.[11]

The situation becomes even more complex when three particles are entangled. Thus, in mathematical terms, a general pure state of three spin-$\frac{1}{2}$ particles, $|\Psi_{gen}\rangle$, now has to be written as the superposition of eight possible states, viz, one with all the spins pointing up, and one with them all pointing down; three in which two of the spins point up and one points down, and three where two of the spins point down and one points up, ie

$$|\Psi_{gen}\rangle = a|\uparrow\uparrow\uparrow\rangle + b|\uparrow\uparrow\downarrow\rangle + c|\uparrow\downarrow\uparrow\rangle + d|\downarrow\uparrow\uparrow\rangle + e|\uparrow\downarrow\downarrow\rangle + f|\downarrow\uparrow\downarrow\rangle + g|\downarrow\downarrow\uparrow\rangle + h|\downarrow\downarrow\downarrow\rangle \quad (2)$$

where as before a, b, c, d, e, f, g, and h are all complex numbers such that the sum of the squares of their real parts $|a|^2+|b|^2+|c|^2+|d|^2+|e|^2+|f|^2+|g|^2+|h|^2 = 1$, and the ket $|\uparrow\uparrow\uparrow\rangle$ represents, for example, the state where all three spin-$\frac{1}{2}$ particles spins are pointing up. This ultimately leads to a total of eight (2^3) three-particle maximally entangled states called GHZ states. For physical reasons, the pair, $1/\sqrt{2}$ $(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)$, is readily distinguishable from the other six.

It is possible to quantify the entanglement between particles in terms of their 'tangle', defined as a number between 0 and 1.[9] For three particles, A, B, and C, there is a simple relationship expressed as follows:

$$\tau_{A(BC)} = \tau_{AB} + \tau_{AC} + \tau_{ABC} \quad (3)$$

which in words reads, 'the tangle of particle A with the rest of the system (B and C) is equal to the tangle of particle A with B alone, plus the tangle of A with C alone, plus the three-way tangle of the whole system.' In the case of the GHZ states $1/\sqrt{2}$ $(|\uparrow\uparrow\uparrow\rangle \pm |\downarrow\downarrow\downarrow\rangle)$, each particle is fully entangled with the rest of the system (eg, $\tau_{A(BC)} = 1$), and the three-way tangle, τ_{ABC}, is also equal to 1, *but no two of the particles are entangled with each other*. Thus, equation (3) actually reads 1 = 0 + 0 + 1, so that *three-way entanglement exists only at the expense of pairwise τ_{AB} and τ_{AC} entanglement*. We shall see the importance of this later. Meanwhile, entanglement can be pictorially represented by a set of what are known as Borromean rings[12] interlocked in such a way that if one of them is cut, all three fall apart (the physical equivalent of 'United we stand, divided we fall') (Figure 1).[13]

I will use this kind of three-way entanglement based on the GHZ state, to model the homeopathic interaction between the patient, the practitioner, and the remedy, ie, PPR entanglement.

Weak quantum theory

Introduction

As an explanation of the material world, quantum mechanics is probably the most successful theory ever developed. Part of this success is due to the perceived confinement of quantum theory within a narrow but

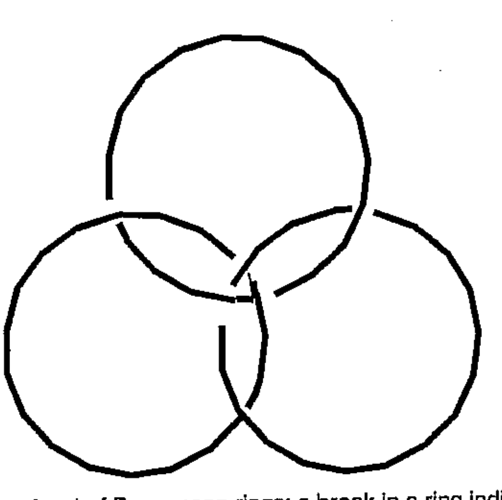

Figure 1 A set of Borromean rings: a break in a ring indicates that it passes under the ring that crosses it. If any one ring is cut, the whole unit falls apart.

extremely important domain: the nanoscopic world of atoms and molecules.

At the very heart of quantum theory lie the notions of complementarity and Heisenberg-style uncertainty relations. In other words, precise knowledge of complementary variables, such as position and momentum, or energy and time, is impossible within the same measurement. Mathematically, this is usually expressed in the form of an algebra of non-commuting operations, ie, it is crucially important in what order measurements of complementary variables are made. In classical mechanics, there is no such uncertainty and the operations describing simultaneous measurements of observables, eg, the position and momentum of a speeding car, do indeed commute (commutation is best understood by considering simple multiplication. Thus 3 times 2 is the same as 2 times 3. The mathematical operation of multiplying integers is a commuting operation, the order in which it is done does not matter. Non-commuting algebras are used when it does matter in what order operations are performed, as in cookery).

Although simultaneous knowledge of complementary variables is impossible, both are required in order to provide a holistic account of reality. This underpins entanglement and the non-local (ie, unsignalled and simultaneous) or global correlations between the elements of a system, which Einstein so deplored, even when they are separated by vast tracts of space and time.[14]

Returning to the algebra of non-commuting operations, this can be formally expressed as

$$|A, B| = |AB| - |BA| = iC \qquad (4)$$

where A and B represent a pair of non-commuting operations (eg, position and momentum or energy and time) and C is called the *commutator* (in commuting classical physics operations, $C = 0$). The reason why the scope of quantum theory is perceived to be confined to the nanoscopic domain is because in the fundamental relation shown in equation (4), the commutator = $h/2\pi$, and h is an extremely small number, 6.626×10^{-34} J/s, ie,

$$|A, B| = |AB| - |BA| = ih/2\pi \qquad (5)$$

Extending the applicability of quantum theory

Notions of complementarity and entanglement have implications far beyond the specific meaning ascribed to them by quantum theory.[8] Examples (admittedly using far less formal approaches than quantum theory) have been cited from engineering and the cognitive sciences, especially psychology (see reference 8). Perhaps one of the most profound applications of complementarity and entanglement concerns the relationship between mind and matter, or more precisely, the mental and material observables of a system. Jung and Pauli[15] explored this in a very broad sense, speculating that behind these epistemological (and complementary) mental and material domains lies a deeper ontological reality, destroyed by the 'Cartesian split', and that the so-called synchronistic correlations between mind and matter remain as remnants of this lost wholeness. We shall see how this complementarity of mind and matter has a direct bearing on homeopathy.

In such ways ordinary quantum theory is being applied outside of its usual physical context,[16] but Atmanspacher, *et al.*[8] take a more radical approach. They generalise the standard quantum theoretical framework by relaxing the conditions used in ordinary quantum physics, so that complementarity and entanglement become useful concepts in much broader contexts. This relaxed, more generalised version of ordinary quantum theory is called WQT. WQT shares with ordinary quantum theory complementarity and non-commutability of observables. Also in WQT, as with ordinary quantum theory, holistic correlations and entanglement arise if, in a system consisting of many parts, observables pertaining to the whole system are incompatible with observables of its parts.

WQT differs fundamentally from ordinary quantum theory in three ways.

- Complementarity and entanglement are not restricted to a particular *degree* of non-commutativity of observables. In other words, there is no constant like Planck's constant (h), in WQT so equation (5) does not apply. Rather, the degree of non-commutativity will vary from case to case.
- WQT has no interpretation in terms of probabilities, as does ordinary quantum theory, and expectation values (ie, the mean value of a series of determinations of an observable) corresponding to operators[17] are non-linear, not linear as in ordinary quantum theory.
- In WQT complementarity and indeterminacy are not ontological (ie, belonging to the very nature of the system and cannot be decomposed by refinement of observation) as in ordinary quantum theory rather they are of epistemological origin (ie, what can be known about the system by observation). In WQT it is much easier to argue that incomplete knowledge of a system or perturbations caused by observation are epistemological reasons for complementarity and indeterminacy.

Finally, it is worth pointing out that, from the point of view of the theory of categories (ie, the 'mathematics of mathematics') WQT is a much more basic and general theoretical description of nature than quantum mechanics: the latter is really a special case of WQT.

Refining the quantum metaphor for homeopathy

We can now refine the quantum metaphor for homeopathy and PPR entanglement by applying the above theoretical concepts. In the previous paper,[3] it was pointed out that the metaphor was necessarily crude because MQT, on which it was based, effectively ignores entanglement. In continuing the search for better quantum metaphors of PPR entanglement, the GHZ quantum entangled state appears to offer a better model. The non-commuting algebra of ordinary quantum theory severely restricts its applicability to the nanoscopic domain, but the more generalised formalism of WQT lifts some of those restrictions, potentially allowing quantum theory applications outside the narrow domain of particle physics.[8]

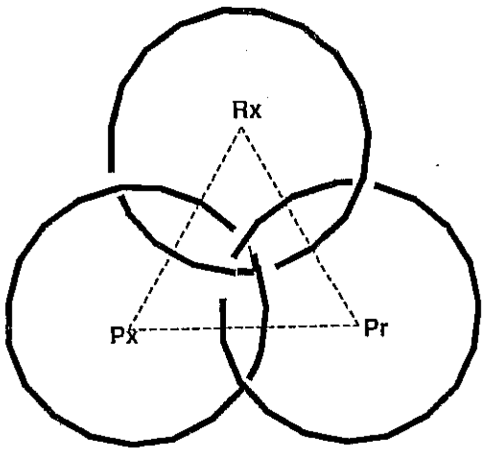

Figure 2 PPR entanglement seen as a set of interlocking Borromean rings.

The GHZ state and PPR entanglement

In order for PPR entanglement to be describable in terms of something like the GHZ state, $1/\sqrt{2}(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)$, it is necessary to find equivalents of the 'up' and 'down' spin orientations of three entangled particles, or the horizontal and vertical polarisations of photons, to describe the different states of the patient Px, the practitioner Pr, and the remedy Rx. Clearly, compared to particles or photons, each of the latter can exist in a multitude of states, but for the purposes of argument I shall restrict my choice of states to just two for each of the entities, Px, Pr, and Rx. In addition, the states of quantum entities are imagined to exist in what is known as a 'space of states'. For quantum entities, this space is called Hilbert space. I shall therefore imagine that the 'states' of the entities Px, Pr, and Rx exist in a similar space of states 'arena', called a 'therapeutic state space'.

Thus for any potentially therapeutic situation existing within such a therapeutic space of states, the patient Px may be considered to be in a state of wellness ($|Px\uparrow\rangle$) or un-wellness ($|Px\downarrow\rangle$), the practitioner Pr to be helpful ($|Pr\uparrow\rangle$) or unhelpful ($|Pr\downarrow\rangle$), and the remedy Rx to be curative ($|Rx\uparrow\rangle$) or non-curative ($|Rx\downarrow\rangle$). Thus, by analogy with the GHZ state, the PPR entangled state may be represented by

$$|\Psi_{PPR}\rangle = 1/\sqrt{2}(|Pr\uparrow Px\uparrow Rx\uparrow\rangle \pm |Pr\downarrow Px\downarrow Rx\downarrow\rangle), \quad (6)$$

What does this mean? The PPR entangled state corresponds to a superposition of Pr, Px, and Rx 'wave-functions', shown in equation (6), leading to a new wave-function $|\Psi_{PPR}$, in the therapeutic state space. The state $\Psi_{PPR}\rangle$ is 'prepared' by the interaction between the Pr, Px, and Rx through the case taken by Pr which results in a holistic picture of the state of Px in the mind of Pr, containing the relevant Rx which matches Px's state of unwellness. In terms of the Borromean rings analogy Figure 2 shows the PPR entangled relationship.

Pr taking and understanding the case and prescribing the Rx to Px, corresponds to all three becoming entangled in a therapeutic state space. The correlation of all three can lead to a response from Px's vital force, leading to a change from unwellness to wellness if the Rx is well chosen. One could describe this process in terms of the transactional interpretation of quantum mechanics,[4] described previously,[2] eg, the Px's experience of symptoms and search for a Pr constitutes the 'offer' wave, while Pr taking the case corresponds to the 'confirmation' wave: the 'handshake' across space–time corresponds to Pr understanding the case and prescribing Rx. Alternatively, one could adopt the more orthodox Copenhagen interpretation of quantum mechanics,[18] and say that the 'operation' (in the mathematical sense) of Pr on Px via the giving of Rx results in a 'collapse' of the entangled-state wave-function Ψ_{PPR}, in the therapeutic state space, leading to a change in the state of Px's vital force from unwellness to wellness, if Rx is well-chosen.

By analogy with the quantification of three-way entanglement shown in equation (3), it should be possible to quantify the degree of PPR entanglement in a therapeutic state space as

$$\tau_{Pr(PxRx)} = \tau_{PrPx} + \tau_{PrRx} + \tau_{PrPxRx} \quad (7)$$

where $\tau_{Pr(PxRx)}$ represents the tangle of Pr with the rest of the system (Px and Rx), τ_{PrPx} represents the tangle of Pr with Px alone, τ_{PrRx} represents the tangle of Pr with Rx alone, and τ_{PrPxRx} represents the three-way tangle between Pr, Px, and Rx. Equation (7) may be interpreted as follows:

- It predicts that the ideal therapeutic situation arises from complete PPR entanglement, in other words, when $\tau_{Pr(PxRx)} = 1$.
- In order for this to occur, $\tau_{PrPx} = 0$, ie, Pr should not entangle with Px alone, which could be taken to mean that the practitioner should not over-identify with the patient.

- It also means that $\tau_{PrRx} = 0$, ie, Pr should not entangle with Rx alone, which, in turn could be taken to mean that the practitioner should not over-identify with 'finding the right remedy'
- If these three conditions are met then, $\tau_{PrPxRx} = 1$, ie, Pr entangles completely with both the patient and the remedy.

These four points, in my view, imply that Pr has to tread a very fine line: in order for Px to gain maximum benefit from the entangled PPR state, Pr must beware of not over-identifying with the patient or become too concerned about finding the correct prescription. In terms of Pr's state of mind during the therapeutic encounter, the GHZ/PPR entangled state implies that Pr's awareness would need to be directed inward as well as outward as the encounter progresses, in order to ensure the maximum three-way entanglement and the minimum two-way entanglements (as described above). Certainly, such Janus-like states are not unknown within the psychotherapeutic domain, where Pr has to be aware of his/her own inner state during the consultation, to be on guard for the possibility of counter-transference phenomena from the patient.[8]

WQT and PPR entanglement

As noted earlier, ordinary quantum mechanics differs from WQT in that in the latter, complementarity and entanglement are not restricted to a particular *degree* of non-commutativity of observables: there is no constant like Planck's constant in WQT so equation (5) does not apply. Rather, the degree of non-commutativity will vary from case to case, so that a more realistic equation describing the non-commutativity of observables would be like equation (4), eg,

$$|A, B| = |AB| - |BA| = iC \qquad (4')$$

The question then is what could the observables A and B, and the commutator C correspond to in the homeopathic situation? A possibility in line with the GHZ model of PPR entanglement is that A corresponds to observables $|Px|$ about the patient made by Pr, while B corresponds to other observables $|Pr|$ (see below) in the therapeutic encounter, also made by Pr. The commutator C, could then correspond to the remedy, Rx, leading to the non-commuting relationship

$$|Px, Pr| = |PxPr| - |PrPx| = iRx \qquad (8)$$

Here, the commutator Rx would be different for each therapeutic situation. Equation (8) implies that:

- Pr is not only making ***local*** observations about the patient, $|Px|$ (which become melded into a holistic picture of the patient's symptom totality, $Sx = \Sigma Sx_i$ representing a set of ***local*** observables), but;
- Pr is also making ***global*** observations $|Pr|$ which could include general observations about the Px–Pr interaction, eg, self-monitoring of Pr's own inner state and how it fluctuates in response to Px (this may be considered ***global*** because ultimately, we are all the centre of our own universe).
- Both ***local*** and ***global*** sets, $|Px|$ and $|Pr|$ represent an orthogonal pair of complementary observables related by the non-commuting algebra represented in equation (8), which ultimately leads to the choice of homeopathic remedy, Rx.
- Thus, equation (8) could be said to represent a non-commuting algebra of the homeopathic process.

It is worth making one other point about the algebra of the PPR entangled state. In quantum theory, experimental observations are described by operators and there is a connection between wave-functions, operators (and the observations associated with them), and the outcomes of measurements, leading to what are called 'expectation values'.[19] An expectation value may be defined as equal to the mean value of an observable in a series of experimental measurements.

The expectation value of an operator Ω, is denoted by $\langle\Omega\rangle$. For an arbitrary state described by the wave-function ψ, the expectation value is defined by

$$\frac{\int \psi * \Omega \psi \mathrm{d}t}{\int \psi * \psi \mathrm{d}t} = \langle\Omega\rangle \qquad (9)$$

where ψ^* is the complex conjugate of the wave-function ψ.[10] If the wave-function is chosen to be normalised to 1, ie, $\int \psi^* \psi \, \mathrm{d}t = 1$ or, using Dirac notation, $\langle\psi|\psi\rangle = 1$, then equation (9) becomes

$$\langle\psi|\Omega|\psi\rangle = \langle\Omega\rangle \qquad (10)$$

where the bra $\langle\psi|$ represents the complex conjugate of the ket $|\psi\rangle$.

A similar equation could be written to describe the PPR entangled state produced during the homeopathic process. Thus, substituting in equation (10), ψ becomes the normalised PPR entangled state wave-function Ψ_{PPR}, the operator Ω, becomes the 'homeopathic operator' Πr, which operates on the PPR entangled wave function and its complex conjugate to produce the 'expectation value' for the process, which is the remedy $\langle Rx\rangle$:

$$\langle\Psi_{PPR}|\Pi r|\Psi_{PPR}\rangle = \langle Rx\rangle \qquad (11)$$

The expression $\langle\Psi_{PPR}|$ is said to be the 'bra' of Ψ_{PPR} and mathematically represents the complex conjugate of the ket $|\Psi_{PPR}\rangle$.[11] The question then is what could the notion of 'complex conjugate' mean in terms of this model. For this, we need to understand a little more about complex numbers, eg, $x + iy$, and their relationship with their complex conjugates (see appendix and footnote[†]).

In equation (11), the PPR entangled state wave-function Ψ_{PPR} is normalised and the coefficient $1/\sqrt{2}$ appears in the GHZ formalism (see equation 6). Thus the absolute values of the coefficients contained in the PPR entangled state wave-function Ψ_{PPR} must be equal to each other. It follows that the complex conjugate of Ψ_{PPR} can be obtained simply by multiplying it by –i, ie, $\Psi_{PPR}^* = -i\Psi_{PPR}$ or, rewriting this in Dirac 'bra' and 'ket' notation, $\langle\Psi_{PPR}| = -i|\Psi_{PPR}\rangle$. Just as the complex number $x + ix$ and its conjugate $x - ix$ are reflections of each other in the x-axis of the complex plane (see appendix), so $\langle\Psi_{PPR}|$ and $|\Psi_{PPR}\rangle$ represent the mirror-like quality of health and disease in a therapeutic state space, as experienced and expressed by Px, ideally reflected back by Pr and affected by Rx. This is what is expressed in equation (11) where the **Πr** operator-mediated relationship of $\langle\Psi_{PPR}|$ and $|\Psi_{PPR}\rangle$ results in the remedy **Rx**.

In a further refinement, which takes into account one of the main differences between WQT and ordinary quantum theory (ie, that expectation values corresponding to operators are if anything non-linear), equation (11) can rewritten as

$$\langle\Psi_{PPR}|\mathbf{\Pi r}|\Psi_{PPR} = \langle(\mathbf{Rx})^p\rangle \qquad (12)$$

where the index p represents Rx raised to some power. What does this mean in the context of the homeopathic process? The non-linearity expressed in this equation could have something to do with the potency (hence the superscript 'p') of the prescribed remedy, presumably the index p being included with Rx, as an outcome of PPR entanglement.

Finally, there is the question of 'how long' the PPR entangled state could persist. Quantum entangled states are known to last for no more than about half a millisecond, and unequivocal experimental evidence for non-quantum entangled states has yet to be convincingly demonstrated. By analogy, one could suggest that PPR entanglement occurs during case-taking and 'ends' when the Pr 'gets' the case and decides on a remedy (in the Copenhagen Interpretation of quantum mechanics, this would be the equivalent of 'collapsing' the Ψ_{PPR} wave-function to the point of cure: if so, then 'cure' should 'begin' at this point, possibly even before Px has physically taken the Rx).‡ The Transactional Interpretation of quantum mechanics[2,3] as a quantum metaphor effectively avoids the problems with causality thrown up by this approach. However, 'when' during the case-taking PPR entanglement 'begins' is, from the point of view of the GHZ/WQT treatment described in this paper, a matter of concern for Pr. In order to maximise three-way entanglement and minimise two-way entanglements, Pr would have to 'prepare' an inner state in which attention is simultaneously directed outwards (for local observations about Px) and inwards (for global observations about Pr and the Px–Pr interaction). Such states are not easily prepared, and unless Pr is well-practiced in the art, neither can they long be maintained.

Summary

James Tyler Kent wrote that 'a remedy is only homeopathic when it cures the case',[20] seeming to imply that the remedy cannot be considered in isolation from the practitioner who prescribes and the patient who takes it. In the terminology of quantum theory, Kent could be suggesting a kind of entanglement between patient, practitioner, and remedy.

The same may be said about disease. A petri dish containing a living culture of methicillin resistant *Staphylococcus aureus* (MRSA) bacteria is not a disease until it infects a patient and is diagnosed by a practitioner. Thus, there is an intimate correlation (entanglement) between diseased patients, remedies, and practitioners. Conventional medical science has attempted to factor out all these essentially entangled entities into clear-cut separate (and empirically testable) elements. The result of this reductionist programme is the increasing emphasis on molecular pharmacology (at the expense of the human interaction between the patient and the practitioner) as the only factor really worthy of consideration in disease processes.[21]

This is not to say that molecular pharmacology is wrong, or that the 'memory of water'[22] might not be a plausible pharmacological explanation of how homeopathy 'works': far from it. However, by concentrating only on local interactions (eg, the spatial-temporal interactions of small drug molecules with cellular 'receptor sites', or the putative memory-of-water-mediated pharmacological action of the highly attenuated remedy as used by homeopaths) as the sole meaning of 'cure', the more global (ie, non-spatial and atemporal)[1] interactions between patient and practitioner, and their influence on the curative process, are effectively ignored.

PPR entanglement, as discussed here and previously, is an attempt to reinstate the importance of these global interactions in the therapeutic process by the use of quantum theoretical metaphors. It has been the aim of this paper to begin the construction of an algebra for the therapeutic process that describes the entangled state of the patient, the practitioner and the remedy. To this end, the GHZ state of quantum theory has been used to model the PPR therapeutically entangled state, which may be quantifiable by analogy with the three-particle entanglement of the GHZ state. Thus, for each of the Px, Pr, and Rx to be fully entangled, no two of these entities should be individually entangled

‡I have commented on this in an earlier paper (see reference 2) where the transactional interpretation of quantum mechanics is used. Here, the PPR entangled interaction is imagined as culminating in an atemporal 'handshake' between Px and Pr that corresponds to Pr understanding the case and prescribing the Rx. The atemporal nature of this process by-passes problems with causality, such as those admittedly anecdotal reports of patients reporting improvement in their symptoms before they have received the potentised Rx (see reference 2 and references therein).

with each other. Then, WQT was used to provide the basis of a non-commuting algebra for the homeopathic process. The commutator is predicted to be the remedy Rx, potentially different for each case. A basic equation relating the wave-function of the PPR entangled state to the operation of the practitioner and the remedy as an expectation value, was also hypothesised. It was also suggested that health and disease could have a mirror-like relationship similar to and represented by the relationship of complex numbers to their complex conjugates (note: wave-functions are constructed using complex numbers). Finally, some speculations were presented about how long the PPR entangled state might persist and Pr's possible role in initiating this state by dividing attention between Px and his/her own inner state. This is only the beginning of a long process: further work may expand these concepts, hopefully leading ultimately to new ways to experimentally verify homeopathy.

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Appendix

Complex numbers may be exhibited on a plane in which the horizontal coordinate x represents the real part of the complex number, while the vertical coordinate iy represents its imaginary part (although y itself is a real number). The two axes define the complex plane.

Thus in Figure 3, the real part of the complex number can be read off from the horizontal axis as x,

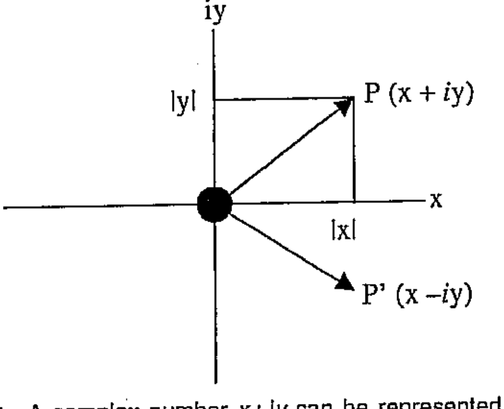

Figure 3 A complex number $x+iy$ can be represented on a plane in which the horizontal coordinate represents the real part of the complex number while the vertical axis represents the imaginary part.

while the imaginary part can be read off the vertical axis as iy. The complex conjugate of $x + iy$ is $x - iy$ which can be obtained by reflecting the line joining the origin O to point P in the x-axis: this gives the line OP'. Thus in this formalism, a complex number and its conjugate are mathematical mirror reflections of each other in the horizontal or real axis.

The real and imaginary axes in Figure 3 divide it into four quadrants. Multiplication of a complex number by i ($= \sqrt{-1}$ and $i^2 = -1$) translates it anti-clockwise into the next quadrant. Thus, $x + iy$ multiplied by i $= ix - y$: multiplying again by i $= -x - iy$: and again by i $= -ix + y$. Now, if the complex number is normalised, that is the square of the absolute values of the complex number's coefficients x and y are equal to 1 (ie, $|x|^2 + |y|^2 = 1$) and $|x| = |y|$, then by definition, $|x| = |y| = 1/\sqrt{2}$ and the complex conjugate of $x + ix = x - ix = -i(x + ix)$. This means that, assuming the above restrictions, the complex conjugate can be obtained simply by multiplying a complex number by $-i$. As wave-functions are constructed using complex numbers, it follows that a normalised wave-function can be converted into its mirror image complex conjugate also by multiplication by $-i$.

A mathematical function is said to be factorisable if it can be separated into parts. Consider, for example the general quadratic function, $ax^2 + bx + c$. If the coefficients a, b, and c have the right values, then the quadratic function will factorise, eg, when $a = 1$, $b = -4$, and $c = 3$; the quadratic function $x^2 - 4x + 3$ factorises into a product of two simpler linear functions, $(x - 3)(x - 1)$. On the other hand (and disregarding complex numbers for the moment), when $a = 1$, $b = 1$, and $c = 1$, the quadratic function $x^2 + x + 1$ does not factorise or break down into a product of two simpler linear functions.

Though much more complicated than quadratic functions, electronic wave-functions (which are constructed using complex numbers) can be treated in a similar mathematical fashion. Thus, when two electronic wave-functions ψ_A and ψ_B are combined, the resulting new electronic wave function Ψ describes an entangled state of the two electrons if it cannot be factorised or broken down back to its component simpler wave-functions. This means that the new wave-function Ψ actually describes a single non-classical indivisible whole and we are forced to consider that what ever is done to one of the electrons instantaneously (ie, faster than the speed of light, atemporally and non-spatially) affects its entangled partner. Such is the strangeness of the quantum world. On the other hand, if the new wave-function Ψ is factorisable back into ψ_A and ψ_B, then it is simply describing two un-entangled classical entities.